Key

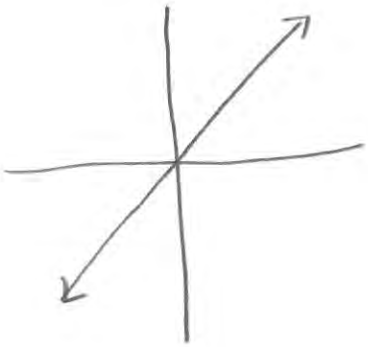
Algebra II Key features of function

**Objective**: Identify x and y intercepts, domain, range, increasing, decreasing, maximum minimum, positive and negative and average rate of change for functions and their graphs.

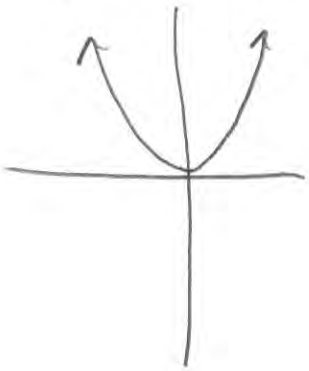
**Warmup: Explore and reason.**

**There are 4 basic functions we are going to work with today. Here are the graphs.**

**y=x** linear slope of 1

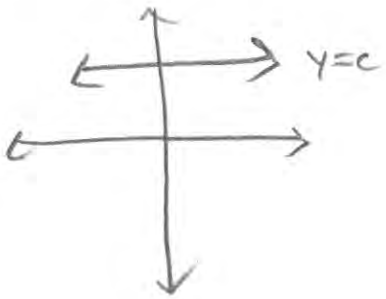


**$y=x^2$** parabola



**y=|x|** V

**y=c where c is any number** horizontal




**Interval notation**: uses x values to represent the interval (like an inequality $a \le x \le b$ )

( or ) to not include a point (open circle) $<$ (2,3) means $2 < x < 3$

[ or ] to include a point (closed circle) $\le$ [2,3] means $2 \le x \le 3$

∞ always uses (2,∞) means $x > 2$ (−∞,2) means $x < 2$

Practice. Write in interval notation.

| $3 \le x \le 4$ | $3 < x < 4$ | $3 < x \le 4$ | $3 < x$ | $x \le 4$ |
|---|---|---|---|---|
| [3,4] | (3,4) | (3,4] | (3,∞) | (−∞,4] |
| | | | all points greater than 3 | ↑ includes all points less than or equal to 4 |

left-right on graph

**Domain:** all values for x (independent variable)

**Range:** all values for y (dependent variable)

lowest-highest

Algebra II Key features of function

left, right ↑ lowest, highest ↑ ∪

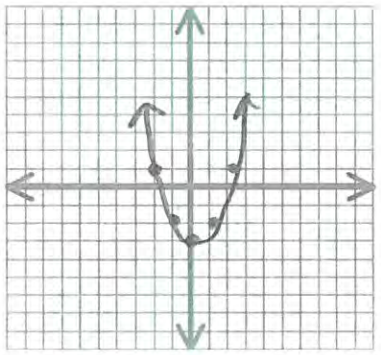
**Exampe 1. Domain and range:** Find the domain and range of $y=x^2-3$.

Domain: all x $(-\infty,\infty)$

Range: y greater than or equal to $-3$ $[-3,\infty)$ $y \geq -3$

| x | y | |
|---|---|---|
| -2 | 4-3 | 1 |
| -1 | 1-3 | -2 |
| 0 | -3 | |
| 1 | | -2 |
| 2 | | 1 |

$\{x \mid x \in \mathbb{R}\}$ $\{y \mid y \geq -3\}$



An airtanker flies over forest fires and drops water at a constant rate of 400 g/sec until his 8000 gallon tank is empty. What is the domain and range of the function that represents the volume of water the airtanker can drop in x seconds. rate

y = gallons
x = time

$y = 400x$

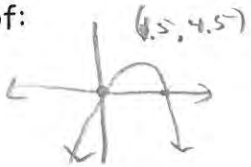
$8000 = 400x$
$20 = x$
20 seconds until gone

Domain $[0,20]$
Range $[0,8000]$

You try. What is the domain and range of:

$y=|x-4|$
D: $(-\infty,\infty)$
R: $[0,\infty)$

$y=6x-2x^2$



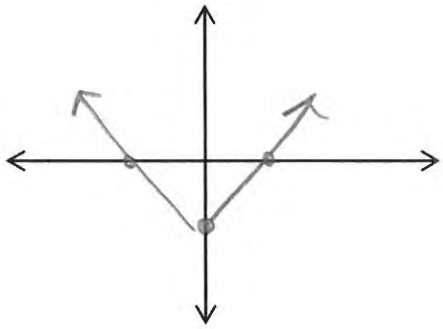

D: $(-\infty,\infty)$
R: $(-\infty,4.5)$

**Key features fo functions**

X intercept (zero): cross the x axis ; make y=0 and solve

Y intercept: cross the y axis
make x=0 and solve

**Example 2.** Find the x and y intercept of $y=|x|-3$.



x=0 $y=|0|-3$
$y=-3$ → y int. (0,3)

y=0 $0=|x|-3$
$3=|x|$
$\pm 3=x$ → x intercepts (3,0) (-3,0)

Algebra II Key features of function

Can a function have more than one x intercept? Give an example or counterexample. yes just saw

Can a function have more than one y intercept? Justify your answer. no it would not be a function

A car starts a journey with a full tank of gas. The equation y=16-.05x represents the number of gallons of gas , y, left in the tank after the car has travelled x miles. What are the x and y intercepts and what do they mean?

$y=0 \quad 0=16-.05x$

$-16=-.05x$

$320=x$

when 0 gallons, travelled 320 miles (out of gas)

$x=0 \quad y=16-0=16$

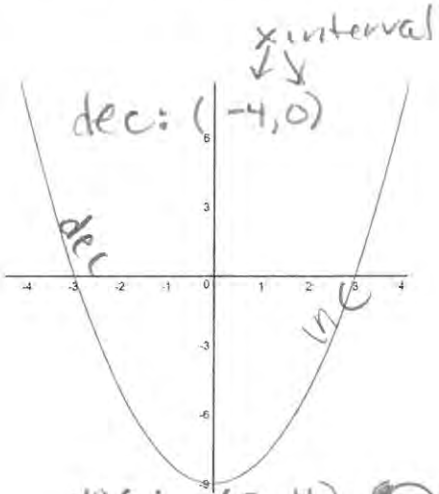
When 0 miles, I started with 16 gallons.

Determining **intervals** of increasing and decreasing.

A function is **increasing** when: slope + ↗

A function is **decreasing** when: slope − ↘

A function is **constant** when: slope = 0 ↔




Determine **intervals** where the function is positive or negative.

A function is **positive** when: above the x axis

A function is **negative** when: below the x axis

**Example 4**. Determine the intervals where the function f(x)= 2-|x| is increasing, decreasing, or constant, positive or negative. Also state domain, range, and intercepts.

Increasing: $(-\infty, 0)$

Decreasing: $(0, \infty)$

Constant: never

Positive: $(-2, 2)$

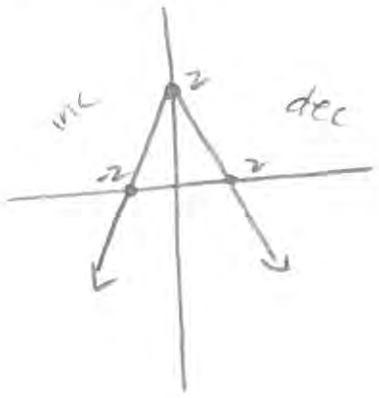
Negative: $(-\infty, -2) \cup (2, \infty)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 2]$

X intercepts: $(-2, 0)$ $(2, 0)$

Y intercepts: $(0, 2)$



Algebra II Key features of function

**Note:** use **interval notation** for domain, range, increasing, decreasing, constant, positive or negative. All are given in terms of x (the independent variable) except for range.

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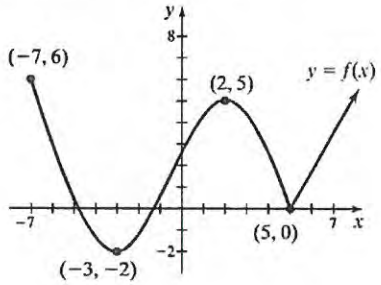
**Average rate of change.**

The average rate of change is equal to the slope between any two points on the graph.

Find the average rate of change between x= -7 and x=2.

(-7,6) (2,5) $\frac{5-6}{2-(-7)} = \frac{-1}{9}$




Find the average rate of change between x=-3 and x=5.

(-3,-2) (5,0) $\frac{0-(-2)}{5-(-3)} = \frac{2}{8} = \frac{1}{4}$

**Putting it all together.** Find the information based on the graph.

+ slope Increasing: (-6,2) (8,11)

- slope Decreasing: (2,8)

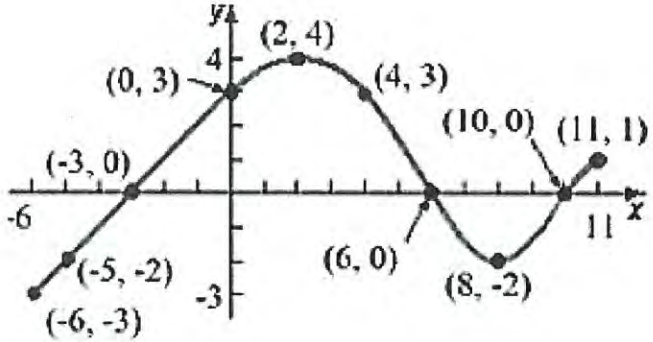
Constant: —

above x Positive: (-3,6) ∪ (10,11)

below x Negative: (-6,-3) ∪ (6,10)

L-R Domain: [-6,11]

b-Top Range: [-3,4]




X intercepts: (-3,0) (6,0) (10,0) Y intercepts: (0,3)

Avg rate of change from x=-5 to x=4 (-5,-2) (4,3) → $\frac{3-(-2)}{4-(-5)} = \frac{5}{9}$

f(0)= 3 point

f(8)= -2

f(-6)= -3

For what values of x is f(x)=3? when is y=3?

x=0 x=4